Mechanisms of the Autonomous Control

Application to Short- and Long-Term Problems of Control the Economic Growth

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Abstract

Two options of models of mechanisms of an autonomous control being intended for optimization of the current value of criterion function are offered. The feature of models is that the equations of dynamics of object of control aren't used in them. Information about the object is limited to the values of the parameters of control and objective function at the moments of time, which correspond to the current state of object and close to it. These mechanisms assume existence of extremum for the function of criterion, and the nonlinear nature of connections and limitations of the object of control. Conditions for sustainable growth of economic systems with diverse mechanisms of autonomous control are considered.

Keywords

Autonomous Control Mechanisms; Criterion Function; Equations; Dynamic Optimizing Feedback

Introduction

The purposes of simulation of the processes of social and economic development, being oriented on the determination of the parameters of the regime of steady growth, are including:

the study of the qualitative behaviour of economic processes in the economic system,

the study of the influence of different mechanisms of autonomous control on the long-term dynamics of the development of region,

the development of methods and procedures of indicative planning.

In this case it is necessary to consider: different hypotheses of the behavior of agents, the essential influence of random factors.

An example is the model of the region with a diversified group of manufacturing industries, the financial sector, the flow of exports and imports, autonomous control mechanisms of intermediate inputs, investment in fixed assets in industry as well as share allocations from the regional fund for investment.

The particular interest represents the mechanisms of the control optimization being aimed on maximization of the criterion function, for example, the gross release.

The mathematical apparatus for autonomous control uses the mechanism of feedback, oriented for the approximation to an extremum of criterion function. One of the simplest in realization of methods of search of an extremum is the method of the fastest descent. For its application at every step of an algorithm it is necessary to evaluate the gradient (derivative according to the controlling parameter) of criterion function and the length of step, which requires repeated calculation of the values of function close to the current point. In contrast to the computational optimization algorithm, dynamic process does not have a possibility to make trial steps in different directions. Here it can vary only step size depending on the change of the objective function. The parameters choice also can be adjusted in process of development of operated process.

Finite-Dimensional Model of the Mechanism of Autonomous Control

Let's designate: $u(t)$ – control action at the moment of time t; $f(t,u)$ – the criterion function maximized on u during each present moment t. Let this function is of a bounded variation on the argument t, differentiated and convex up on u (at its maximizing on this argument). Optimum control $\bar{u}(t)$ satisfies to a necessary condition

$$\partial f(t,u)/\partial u = 0 . \qquad (1)$$

We will decompose the criterion function in a Taylor row of the 2nd order in the fixed point $u(t_i)$

$$f(t,u) = f(t_i, u(t_i)) + \partial f / \partial u(\bar{u} - u(t_i)) + \partial^2 f / \partial u^2 (\bar{u} - u(t_i))^2$$

The necessary condition of an optimality on u for

square-law approach of criterion function in a small vicinity of an optimum point will become

$$\partial f / \partial u + 2\partial^2 f / \partial u^2(\bar{u}-u(t_i)) = 0$$

from where (analogue of a method of Newton) the approach step to an optimum point is defined as

$$\bar{u}(t) = u(t_i) - \frac{\partial f / \partial u}{2\partial^2 f / du^2} \tag{2}$$

The step on optimal control should be combined with a step on time. To consider change of optimum control on time, we will derivate a necessary condition (1) by t.

$$\partial^2 f / \partial u \partial t + \partial^2 f / \partial u^2 \cdot d\bar{u} / dt = 0$$

From where on an optimum trajectory

$$d\bar{u} / dt = -\frac{\partial^2 f / \partial u \partial t}{\partial^2 f / \partial u^2}$$

and the change of optimum control taking into account dependence of criterion function on time is defined as

$$\bar{u}(t) = u(t_i) - \frac{\partial f / \partial u}{2\partial^2 f / du^2} - \int_{t_i}^{t} \frac{\partial^2 f / \partial u \partial t}{\partial^2 f / \partial u^2} dt$$

Thus, the choice of the control at the current step is dependent on the background,

$$\bar{u}(t) = u(t_i) + F(u(t_{i-1}),\ldots,u(t_0), f(t_i,u(t_i)), f(t_{i-1},u(t_{i-1})),\ldots, f(t_0,u(t_0)))$$

Such dependence possesses the following properties when maximizing criterion function near optimum value and at a small step $t - t_i$ on time.

On a trajectory site with optimum control $\bar{u}(t)$ function F accepts zero value.

Out of a site of an optimum trajectory its sign coincides with a derivative df / dt sign.

On a site of an optimum trajectory with accruing control $\bar{u}(t)$ function F is positive.

On a site of an optimum trajectory with decreasing control $\bar{u}(t)$ function F is negative.

Dynamic Optimizing Feedback

Optimizing feedback is aimed at maximizing an criterion $f(t,u)$ in dynamics by a way of a variation of size of the operating parameter $u(t)$. Let's determine this parameter by the mechanism of a proportional and integrated optimizing regulator (В.Б. Гусев, А.И. Егоров). For this purpose for definition $u(t)$ in dynamics we will receive the differential equation which can be included in the system of the equations describing model of operated process, or to use for realization of the mechanism of an autonomous control for real object.

Let's assume besides, convexity of this function on t and u.

On the trajectory approximation $\bar{u}(\bar{t})$ to the maximum point at time $\bar{t}$, if $u(t_i)$ is close to it, in accordance with (2) can be represented by the differential equation

$$\frac{d}{dt}u(t) = h \cdot \frac{\partial}{\partial u} f(t,u)\Big|_{t_i, u(t_i)}$$

with the initial condition $u(t_i)$, where $h = -\frac{1}{2(\bar{t} - t_i)\partial^2 f / \partial u^2}$

(dynamic analogue of a method of the fastest descent).

In a point of a maximum of function $f(t,u)$ by means of a formula of a full derivative it is possible to present a necessary condition of a maximum in a look

$$\frac{d}{du} f(t,u) = \frac{\partial}{\partial u} f(t,u) + \frac{\partial}{\partial t} f(t,u) / \frac{d}{dt}\bar{u}(t) = 0$$

whence

$$\frac{d}{dt}\bar{u}(t) = -\frac{\partial}{\partial t} f(t,u) / \frac{\partial}{\partial u} f(t,u)$$

On a trajectory the private derivative $\frac{\partial}{\partial u} f(t,u)$ is estimated by means of final increments

$$\frac{\partial}{\partial u} f(t,u) \cong (f(t,u) - f(t-\Delta t,u))/(u(t) - u(t-\Delta t))$$

where Δt is time delay (lag).

From here for any point of a trajectory the feedback equation for the operating parameter $u(t)$ will become

$$\frac{d}{dt}u(t) \cong h \cdot \frac{f(t,y) - f(t-\Delta t,y)}{u(t) - u(t-\Delta t)} - s \cdot \frac{u(t) - u(t-\Delta t)}{f(t,y) - f(t-\Delta t,y)} \tag{3}$$

The step h at control of parameters steals up experimentally in the form of exogenous parameter. On a condition of steady growth the condition

$$\frac{\partial}{\partial t} f(t,u) > 0$$

should be satisfied. The assumption that the value of this derivative changes the trajectory is small enough to determine the value of s as a lower bound for the derivative. This factor is selected in accordance with the expected growth rate. The purpose of selection of parameters h and s – receiving the greatest growth on a trajectory of criterion function within an interval [t_0, T]. Here T is the final moment of the considered period.

Computability Conditions

Use of expression (3) at realization of algorithm of an autonomous control demands additional conditions as numerical estimates of derivatives have a considerable error. It results in instability of realization of control process. For increase of stability of process it is possible to use the function limiting a settlement step on control

$$\Delta\tilde{u}(t_i) = \frac{\Delta u(t_i)}{|\Delta u(t_i)|(\beta-\alpha)+\alpha}, \quad 0 \le \alpha \le \beta$$

Where α, β – the parameters of function selected experimentally or on the basis of studying of criterion function. In a situation when change of criterion function $f(u,t)$ on a step of discrete process is great, it is possible to define estimates of an increment of operating function without calculation of the second derivatives with limiting function in the form of a hyperbolic tangent

$$\Delta\tilde{u}(t_i) = m \cdot \tanh(F)$$

where m – the factor of feedback defined experimentally.

Use of the increment of control calculated thus on a process step at successfully picked up parameters allows to receive steady process with the control which is coming nearer to optimum control $\overline{u}(t)$.

The equation (3) in points where $f(t) = f(t+\Delta t)$, has a singularity. To avoid division on 0, at calculations it is necessary to check this condition and in such points the right member of equation to believe equal δ, close to 0. The last speaks that at during $\delta = 0$ the initial moment of time t_0 the right member of equation is equal 0 and as a result on all trajectory it will keep this value, i.e. this mechanism of an autonomous control will appear disabled.

Thus, the equation (3) will be transformed to a look

$$\frac{d}{dt}u(t) = h \cdot D(f(t) - f(t-\Delta t), u(t) - u(t-\Delta t))$$
$$- s \cdot D(u(t) - u(t-\Delta t), f(t) - f(t-\Delta t)),$$

where by definition

$$D(x_1, x_2) = \begin{cases} x_1/x_2, \text{if } x_2 \ne 0 \\ \delta, \text{ if } x_2 = 0 \end{cases}$$

Each of the considered models possesses a certain area of efficiency outside of which process loses controllability. Really, the finite-dimensional model is constructed in the assumption of sufficient proximity of current state to an optimum and rather small step on time.

Harmonization of Mechanisms for Diverse Autonomous Control

A typical problem for economic systems with self-control is that the existing mechanisms of control are characterized by predominantly plans with the short horizon (В.Б. Гусев). They are focused on achieving short-term results - the maximum economic impact of current economic activity. The processes that will occur in the relatively distant future, have been neglected. Thus, investment projects with a payback period of more than 1 - 2 years old often are rejected. Basic funds are operated outside the terms amortization and not updated. In the long term effectiveness of such economic activity is significantly below the potential level possible. On the other hand, an excessive enthusiasm for long-term plans are not sound-expansion of economic activity diverts funds from direct production, which is fraught with bankruptcies, compression and even the termination of production.

The implementation of the productive capacity of the economic system possible on the condition the coordinated functioning of diverse mechanisms of autonomous control.

The foregoing can be illustrated by means of semiquantitative analysis of single-product model of the economic process in discrete time.

Model of Reproduction

Let the output is determined by the direct costs z, material capacity a and power of basic funds w:

$$v(t) = \min(z(t-1)/a, w(t))$$

Direct costs are in turn defined as the need to ensure increased production by a factor of growth x, and availability of funds

$z(t) = \min(v(t)a(1+ x(t)), \max(v(t) - c - u(t),0))$

where c - final consumption, u - costs costs of building funds. The dynamics of discrete-time funds is as follows:

w(t) = w(t – 1)+(u(t-1) b – w(t-1) d)

where b - return on assets ratio, d - retirement rate. We assume added value $f = v - z$ objective function of the economic process.

Short-term autonomous control mechanism is described by the criterion, which is realized in the dynamics of switching circuit optimizes the feedback (А.И. Егоров) with a nonlinear integrator link:

$x\ (t) = x\ (t\text{-}1)\ m\ \ \tanh\ (df\ /dz)$ (4)

where m - the feedback factor, the estimate of the derivative

df / dz = (f(t) - f(t - 1)) / (z(t) - z(t - 1))

Long-term autonomous control mechanism is described by the criterion, which is realized in the dynamics of switching circuit optimizes the feedback from the linear integrating link:

u (t) = max (min (u + k df / du, v-z-c), 0) (5)

where - the feedback factor, the estimate of the derivative

df / du = (f (t) - f (t - 1)) / (u (t) - u (t - 1))

The model verification was carried out by adjusting the constant model parameters: the initial values of state variables, the feedback coefficients, constants, in order to achieve interpreted the behaviour of dynamical variables.

In this case the curve of power by a small amount higher than the curve issues. Costs of creating funds and direct costs are balanced, that ensures sustainable growth mode.

The simulation results

Simulation of economic systems with short-term mechanism ($k = 0$) is shown in Figure 1.

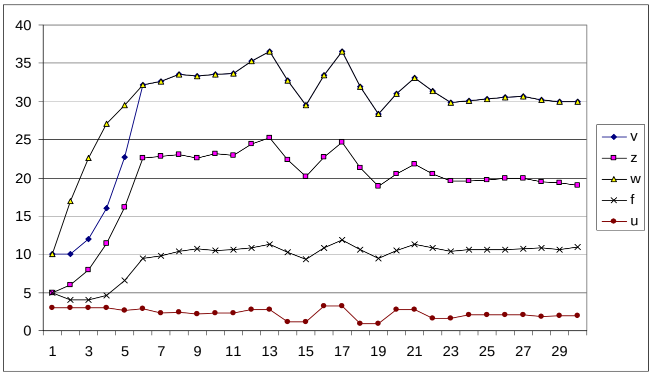

FIG.1. SIMULATION OF ECONOMIC SYSTEMS WITH SHORT-TERM MECHANISM FOR AUTONOMOUS CONTROL

The rapid cessation of growth due to the fact that the increase in direct costs do not leave money on the build-up of basic funds. Simulation of economic systems with long-term mechanism ($m = 0$) is shown in Figure 2.

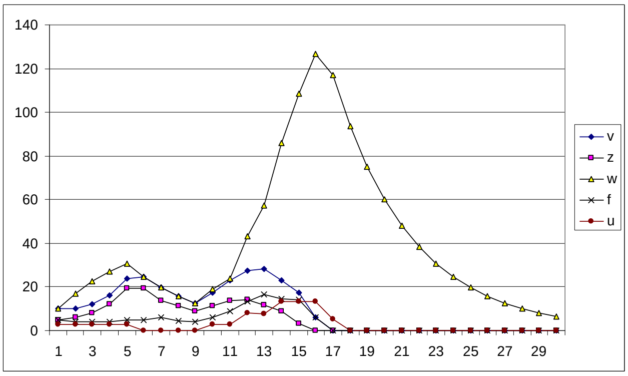

FIG. 2. SIMULATION OF ECONOMIC SYSTEMS WITH LONG-TERM MECHANISM FOR AUTONOMOUS CONTROL

At the initial stage, the situation is developing as well as in the previous case. However, the desire to increase value added by increasing the power dramatically reduces the direct costs.

Simulation of economic systems with agreed mechanisms of autonomous control is shown in Figure 3.

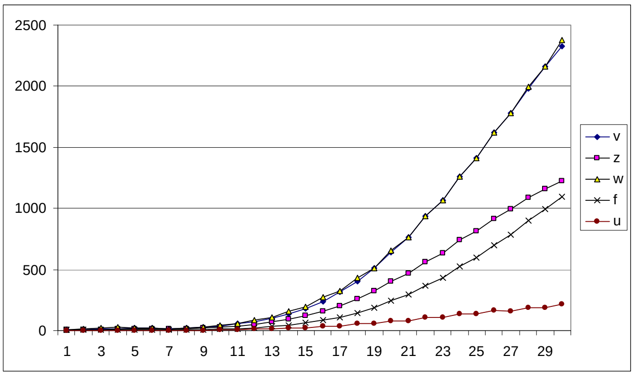

FIG. 3. SIMULATION OF ECONOMIC SYSTEMS WITH AGREED MECHANISMS OF AUTONOMOUS CONTROL

Conclusion

The dynamic model demands control of parameters of feedback that limits its applicability to trajectory sites with a limited variation of criterion function. Numerical experiments show that in such situation at rather small values of a lag Δt the deviation f (t, u) from an optimum also isn't enough.

Inclusion in model of an optimizing regulator allows expanding a range of parameters of model for which the growth mode is carried out.

It is essential that a consistent growth in this model is due only to the simultaneous action of a pair of autonomous control units (4), (5). These units do not use relations of models, are universal and can be implemented in the mechanisms of autonomous control in practice.

The exact formulation of the problem of management and consistent long-term planning requires sophisticated information and organizational support. An important advantage of the mechanisms of management is that their implementation can be performed locally, at the management level and does not require institutional changes in the economic system.

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